Math 10A with Professor Stankova
Quiz 8; Wednesday, 10/18/2017
Section #107; Time: 11 AM
GSI name: Roy Zhao **Name:** ____________________

Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. True **FALSE** When integrating by parts, choosing different functions for u and dv will give you different antiderivatives.

> **Solution:** Now matter which values you choose, you will get the same answer (although they may look different as in the case of integrating $\int \sin(x)\cos(x)dx$).

2. **TRUE** False The error bound for using an approximation method can be 0.

> **Solution:** As we saw in class, if the 4th derivative of f is 0, then your bound for Simpson's method gives 0. This means that the approximation will give the exact answer.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (7 points) Integrate $\int e^{\sqrt{x}}dx$.

> **Solution:** First we u sub to get $u = \sqrt{x}$ so $du = \frac{1}{2\sqrt{x}}dx$ and $dx = 2\sqrt{x}du = 2udu$ Then this integral becomes
>
> $$\int e^{\sqrt{x}}dx = \int 2ue^u du.$$
>
> We can integrate by parts here by setting $r = 2u$ and $dt = e^u du$ so $dr = 2du$ and $t = e^u$. Thus, we get that the integral is
>
> $$2ue^u - \int 2e^u du = 2ue^u - 2e^u + C = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C.$$

(b) (3 points) What is the smallest number of intervals n you need to use in order to guarantee that the trapezoid approximation of $\int_1^2 \ln x dx$ is within $\frac{1}{12\cdot 101}$. (The error bound using trapezoid approximation is $\frac{K_2(b-a)^3}{12n^2}$.

Solution: First we need to calculate $K_2 = \max_{[1,3]} |(\ln(x))''|$. The first derivative is $\frac{1}{x}$ and the second is $\frac{-1}{x^2}$ so the maximum of the absolute value is at $x = 1$ since the function $1/x^2$ is always decreasing. Therefore, $K_2 = |-1/1^2| = 1$. Thus, we have that

$$\frac{1}{12\cdot 101} = \frac{1(2-1)^3}{12n^2} \implies n^2 = \frac{12\cdot 101}{12} = 101.$$

Therefore $n = \sqrt{101}$. But, we want the smallest number of intervals and so we need to take the ceiling. The ceiling of $\sqrt{101}$ is 11 so $N = 11$.